Improving the Usability of HOL through Controlled Automation Tactics

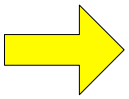
Eunsuk Kang and Mark Aagaard
Waterloo Formal Methods Group

University of Waterloo, Canada

Motivations

- Despite a rich set of available tactics & functions, theorem proving in HOL is not straightforward
- Many user interactions are tedious and repetitive
  - Lots of effort spent on frequent low-level operations (i.e. “doing the grunt work”)
- How can we improve HOL?
  - **Our focus**: Make **common cases** easy & fast

Outline



- **Challenge #1: Choosing Tactics to Use**
  - Solution: Controlled Automation Tactics
- Challenge #2: Working with Assumptions
  - Solution: Extend **`ELIM_TAC`** with Assumption Handling
- Challenge #3: Complexity of Rewriting
  - Solution: Controlled Rewriting Tactics
- Case Studies

Challenge #1: Choosing Tactics to Use

- Main challenge: Too many of them
- “What’s the next step? Which tactic do I apply?”
  - Stiff learning curve for a beginner
  - Non-trivial even for an expert user
- e.g. HOL reference manual – 900+ pages
  - A significant amount of time is spent on browsing through the manual

High-level vs. Low-level Tactics

- **High-level tactics** – “Do as much as you can”
  - e.g. **`PROVE_TAC`**, **`DECIDE_TAC`**, **`RES_TAC`**, etc.
  - Model elimination, decision procedures, resolution, etc.
- Advantage: Completes all or parts of the proof for the user – **if it works**
- Disadvantage: May do more than what user expects (e.g. “things changed magically”)

High-level vs. Low-level Tactics

- **Low-level tactics** – “Do one thing, but do it right”
  - **`CONJ_TAC`**, **`DISCH_TAC`**, **`EXIST_TAC`**, etc.
- Advantage: **No surprises** for the user
- Disadvantage:
  - Tedious to remember names & syntax
  - High-level automated provers (e.g. **`PROVE_TAC`**) are preferred if a goal is easily provable

    (but often hard to tell this just by inspection)

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Our Approach: Controlled Automation

- A **balanced medium** between high-level & low-level tactics
- Controlled automation tactics should:
  - Perform operations **exactly** as the user intends
  - **Give no surprises** - if an operation can't be fulfilled, terminate immediately
  - **Provide improved automation**: Replace existing low-level tactics & remove user's burden of remembering them

Controlled Automation Tactics: Overview

- **GOAL:** Replace most common low-level functions
- Three ideas:

  1. Perform most common logical operations by **guessing "the common thing to do next"**

  2. When a guess requires external guidance, user passes in extra info. through a simple, intuitive mechanism called **hints**

  3. Using **assumption labeling**, let user operate on both the conclusion of the goal and assumptions

Predicting Decomposition Operation

- Observation: Often, we can make a good guess of what user wants to do next by looking at the **outermost logical operator**
- Examples:
  - If the conclusion of a goal is in form `` `P /\ Q` ``, apply **`CONJ_TAC`**
  - If the conclusion is in form `` `∃x. P x` ``, provide a witness to eliminate the existential quantifier

Common Structural Tactics in HOL

- Identify most commonly used operations for goal decomposition
- Replace them with a new tactic called **`ELIM_TAC`**

Operator	HOL Tactics
/\	`CONJ_TAC`
\/	`DISJ_TAC`
==>	`DISCH_TAC`
∃	`EXISTS_TAC`
∀	`GEN_TAC`

ELIM_TAC: Syntax

- Syntax:

```
ELIM_TAC target_label [hint]
```

- **target_label**: a string label for an assumption (empty string "" to refer to the conclusion of a goal)
- **hint**: ML data structure for passing in "extra" information

```
datatype hint =
   ASM of string
 | WITNESS of term
 | INSTANCE of term
 | MATCH of term
 | SPLIT of term
 | SKOLEM;
```

Examples: **ELIM_TAC**

```
Initial goal:
  P /\ Q
By: ELIM_TAC "" [];
2 subgoals:
  Q

  P
```

```
Initial goal:
  P ==> Q
By: ELIM_TAC "" [];
1 subgoal:
  Q                    <- conclusion of goal
  ------------------
  asm. P               <- assumption list
```

Existential Quantifier Elimination

```
Initial goal:
  ∃x. x > 0 /\ x < 2

By: ELIM_TAC?
```

- Next step: Eliminate the existential quantifier using a witness `` `1` ``
- **Problem:** Dangerous to make a guess for witness without user's input
- **`ELIM_TAC`** must somehow receive this extra bit of information

Quantifier Elimination using a Hint

```
Initial goal:
  ∃x. x > 0 /\ x < 2

By: ELIM_TAC "" [WITNESS `1`];

1 subgoal:
  1 > 0 /\ 1 < 2
```

- **WITNESS** (ML type: **term -> hint**) is a constructor of **hint** that specifies the witness for ∃ elimination

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Challenge #2: Working with Assumptions

- Assumption handling is controversial; we think that it is a matter of personal preference
- Depending on one's style of proof, it is useful (and often necessary) to operate on current assumptions

```
Initial goal:
  Q
  ---------------------
  1. P ==> Q
  2. P
```

- In this example: apply modus ponens between two assumptions

Assumption Handling in HOL

- Only a few built-in HOL functions to manipulate assumptions
  - By default, can't point directly to an assumption using a numeric index in HOL
  - Users often end up "hacking" HOL and create their own ML functions to do this

    e.g. **`get_assumption: int -> term`**
- For every structural tactic, user must remember corresponding **forward inference rules**

  e.g. **`DISCH_TAC`** vs. **`MP`**

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Extending `ELIM_TAC` with Assumption Handling

- Extend `ELIM_TAC` to operate on assumptions
  - Same concept as before: **Guess "what to do"** by looking at the structure of an assumption
  - **Uniform** interface for working with **both** the conclusion and the assumptions of a goal
- Instead of numbers, use **user-provided names** to label assumptions
  - \+ Robust to changes in proof
  - -- A few more keystrokes for the user

    (**short-term vs. long-term** gains)

Example: ELIM_TAC

- Previous example on modus ponens

```
Initial goal:
  Q
  ----------------------
  p_imp_q. P ==> Q
         p. P
By: ELIM_TAC "p_imp_q" [ASM "p"];
1 subgoal:
  Q
  ----------------------
  p_imp_q. Q
         p. P
```

- Equivalent to: Eliminate the implication in `` `P ==> Q` `` using assumption `` `P` ``

Another Example: `ELIM_TAC`

- Extract the subterm `` `t2` `` from `` `t1 /\ t2 /\ t3` ``:

```
Initial goal:
  P
  ----------------------
     a. t1 /\ t2 /\ t3
By: ELIM_TAC "a" [];
1 subgoal:
  P
  ---------------------
     a. t1 /\ t2
  a_t3. t3
```

```
Initial goal:
  P
  ----------------------
     a. t1 /\ t2 /\ t3
By: ELIM_TAC "a" [MATCH `t2`];
1 subgoal:
  P
  ---------------------
     a. t1 /\ t3
  a_t2. t2
```

- Users can use hints to specify their **intention**

Making Common Operations Fast

- Some operations on assumptions take **multiple steps**; **`ELIM_TAC`** performs them in a **single step**

Operator	Applied to:	
	Conclusion	Assumption
/\	`CONJ_TAC`	`CONJUNCT1, CONJUNCT2`
\/	`DISJ_TAC`	`DISJ_CASES_TAC, ASM_CASES_TAC`
==>	`DISCH_TAC`	`MP`
∃	`EXISTS_TAC`	`(UNDISCH_TAC + CONV_TAC + GEN_TAC + DISCH_TAC)`
∀	`GEN_TAC`	`SPEC`

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Challenge #3: Complexity of Rewriting

- Challenge: Difficult to control where to rewrite

```
Theorem: ADD_SYM
         [] |- ∀ m n. m + n = n + m
```

```
Initial goal:
  x - (z + y) = x - (y + z)
By: REWRITE_TAC [ADD_SYM];
```

Q. What happens?

A. It will loop forever. By default, **`REWRITE_TAC`** is recursive (i.e. “do as much as you can”)

Rewriting Once in HOL

- Try **ONCE_REWRITE_TAC**, which does not do recursive rewriting

```
Theorem: ADD_SYM
         [] |- ∀ m n. m + n = n + m
```

```
Initial goal:
  x - (z + y) = x - (y + z)
By: ONCE_REWRITE_TAC [ADD_SYM];
1 subgoal:
  x - (y + z) = x - (z + y)
```

- This is still not what we wanted. We want to rewrite only one of the two sides

Specifying Rewrite Location in HOL

- Try **GEN_REWRITE_TAC**, which accepts "conversions" as search strategies:

```
Initial goal:
  x - (z + y) = x - (y + z)
By: GEN_REWRITE_TAC (RATOR_CONV o ONCE_DEPTH_CONV)
    [ADD_SYM];
1 subgoal:
  x - (y + z) = x - (y + z)
```

- **RATOR_CONV**: Applies rewrite only to operator "**f** " of a function application in form, "**f a**"
- **ONCE_DEPTH_CONV**: Applies rewrite only once, using depth-first search

Rewriting Tactics and Conversions

- Lots of built-in rewriting tactics, inference rules and conversions – difficult to remember all these

```
REWRITE_TAC
ONCE_REWRITE_TAC
PURE_ONCE_REWRITE_TAC
GEN_REWRITE_TAC
ASM_REWRITE_TAC
PURE_ONCE_ASM_REWRITE_TAC
ONCE_ASM_REWRITE_TAC
FILTER_ASM_REWRITE_TAC
REWRITE_RULE
… and more
```

```
LAND_CONV
RAND_CONV
RATOR_CONV
ABS_CONV
COMB_CONV
DEPTH_CONV
ONCE_DEPTH_CONV
REDEPTH_CONV
TOP_DEPTH_CONV
… and more
```

- **How can we reduce user's effort in remembering them?**

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Controlled Automation for Rewriting

- **GOAL:** Replace many of the existing rewriting tactics, rules, and conversions
- Two underlying support mechanisms:

  1. Give user a **precise control** over a rewriting location using **hints** - “surgeon’s tool for rewriting”

  2. Using **assumption labeling**, allow user to apply a rewrite rule directly to an assumption or rewrite the goal using assumptions

Controlling Rewriting Location

- Use **higher-order matching** to specify where to rewrite
- Define our own rewrite tactic called **`EQUATE_TAC`**:

```
EQUATE_TAC [hint]
```

- On previous example:

```
Initial goal:
  x - (z + y) = x - (y + z)
By: EQUATE_TAC [MATCH `z + y`, THM ADD_SYM];
1 subgoal:
  x - (y + z) = x - (y + z)
```

- **`MATCH`** specifies a subterm to rewrite & **`THM`** specifies a rewrite rule

Rewriting and Assumptions

- Extend **EQUATE_TAC** to operate on assumptions

```
EQUATE_TAC target_label [hint]
```

- New types of hints & extended roles for existing ones

Hint	Provided to:	
	ELIM_TAC	**EQUATE_TAC**
ASM	=> elimination (modus ponens)	Rewrite rule
ASMS	-	Rewrite rules
MATCH	/\, \/ elimination	Rewrite match
THM, **THMS**	-	Rewrite rule(s)

Controlled Rewriting Tactics

- **Given**: Supporting mechanisms for specifying a subterm & rewriting with assumptions
- **Classify common rewriting tasks** in HOL into three categories:
  - Definitional expansion (**`EXPAND_TAC`**)
  - Term simplification (**`REDUC_TAC`**)
  - Other equational rewriting (**`EQUATE_TAC`**)
- All three tactics share the same syntax as **`ELIM_TAC`**

Comparison of Controlled Rewriting Tactics

	Purpose	Rewriting Behaviour
`EXPAND_TAC`	Automatically expands the definition of a constant stored in the current HOL theory.	Rewrites once.
`REDUC_TAC`	Simplifies a target expression as much as possible using built-in theorems	Repeatedly applies rewrite rules.
`EQUATE_TAC`	Performs general rewriting operations.	Rewrites once.

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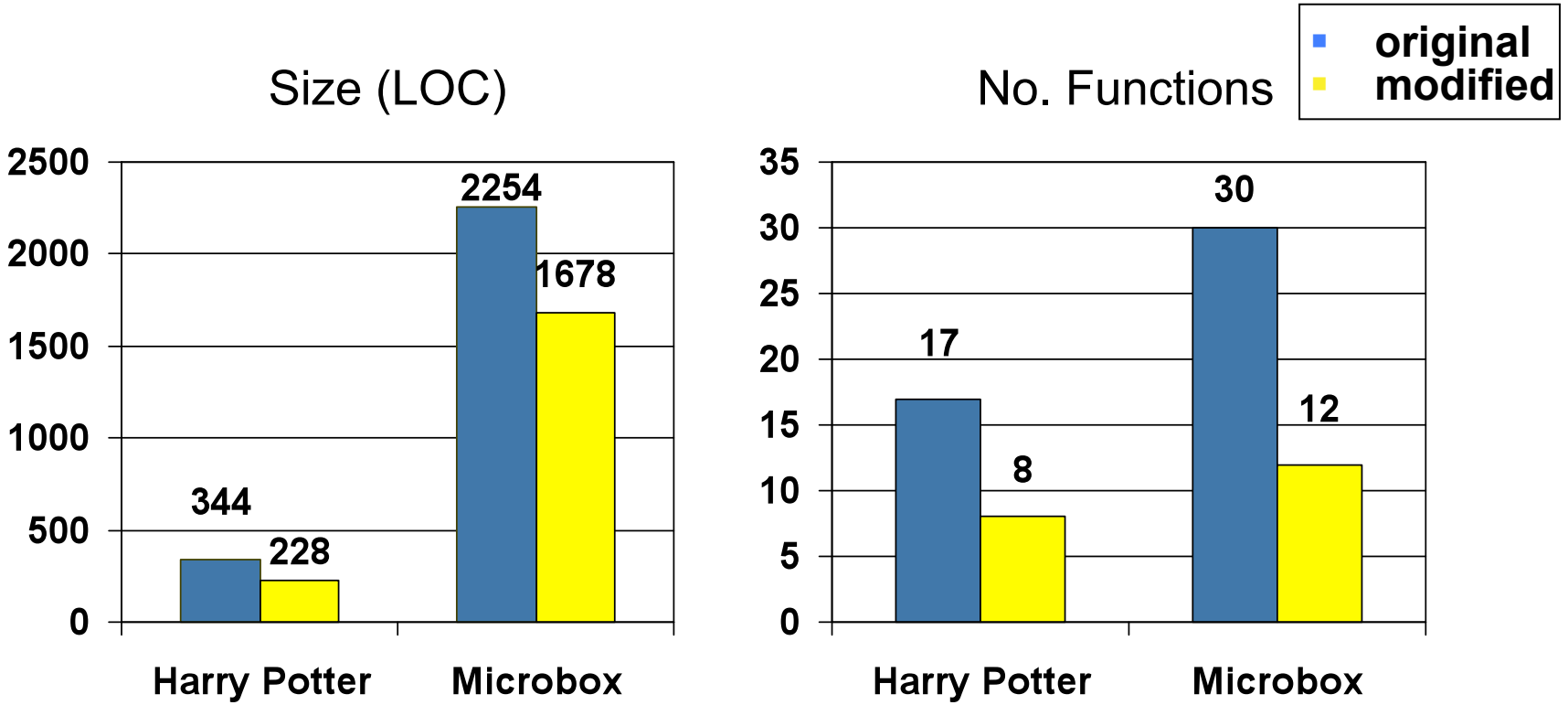
Case Studies

- Used our tactics to re-do two sample proofs
  - A logic puzzle from Harry Potter
  - Proofs about superscalar microprocessor correctness from Microbox project
- Comparison with original proofs:
  - Size of proof script (LOC)
  - Number of unique tactics, inference rules, and conversions used

Case Studies: Results



- Around **25~33%** reduction in the size of proof scripts, and **55~60%** in the number of HOL functions

Case Studies: Remaining Functions

- HOL functions that remained in the modified proof scripts (excluding our controlled automation tactics):
  - **High-level automated provers**: **`PROVE_TAC`**, **`METIS_TAC`**, etc.
  - **Special proof techniques**: **`Induct`**
  - **Tacticals for combining tactics**: **`REPEAT`**, **`THEN`**
  - **Auxiliary functions**: used to add, remove, or rename assumptions
- Most of the low-level tactics/rules have been replaced by the controlled automation tactics!

Summary

- Challenges in improving the usability of HOL:
  - Tactics, assumption handling, and term rewriting
- Our solutions = **Controlled automation tactics**
  - Primitive logical operations: **`ELIM_TAC`**
  - Rewriting operations: **`EXPAND_TAC`, `EQUATE_TAC`, `REDUC_TAC`**
- **Hints**: a simple, intuitive mechanism for user to:
  - Provide our tactics with information required in **precisely** carrying out desired operations
- **Results**: A significant reduction in the number of HOL functions that users need to remember